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A WEIBULL MODEL TO ESTIMATE RESIDUAL AND CRITICAL
VELOCITIES FOR TARGET PENETRATIONS

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J. SPERRAZZA

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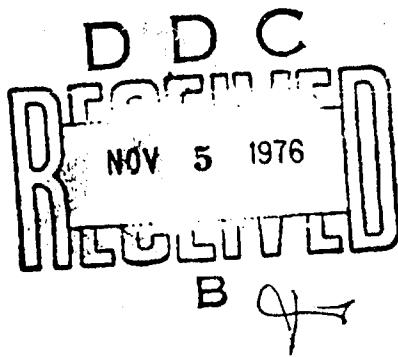
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A procedure based on the versatile three-parameter Weibull distribution function is used to estimate the relationship between the striking velocity and the residual velocity of a projectile penetrating some target material. Nonlinear estimation procedures are used to fit this model to experimental data. A comparison of this model and a hyperbolic model using typical data is presented. A FORTRAN program for implementation of the techniques is included.

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A WEIBULL MODEL TO ESTIMATE RESIDUAL AND
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A WEIBULL MODEL TO ESTIMATE RESIDUAL AND
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D. CLARK
L. CROW
J. SPERRAZZA

1. INTRODUCTION

A number of procedures have been proposed for estimating from test data the functional relationship between V_r, the residual velocity of a projectile after penetration of a target and V_s, its striking velocity. Examples of these are models based on the hyperbolic [1] and exponential [4] relationships

$$V_r^2 = AV_s^2 + B \quad \text{and} \quad V_r = V_s - V_c \, e^{B(1- V_s/V_c)},$$

respectively. In these models, V_c, the critical velocity is defined as the V_s intercept when $V_r = 0$.

Another penetration prediction model is the Johnson equation [2]

$$V_r = (V_s - V_c)[e^{K_4(V_s - V_c)^{K_5}} - 1]$$

where

$$V_c = K_1^{(\sec \theta - 1)}[e^{K_2(A/M \times 10^3)^{K_3}} - 1]$$

and θ is striking obliquity angle in degrees, A is fragment presented area in cm^2, M is fragment weight in grams and K_1, K_2, K_3, K_4, and K_5 are constants to be estimated from the data. For this model the critical velocity is not directly estimated from test data. Instead, the critical velocity is determined by using the estimated V_c's from an empirical model, such as the hyperbolic, fitted to several sets of data to estimate the constants K_1, K_2 and K_3.

For meaningful applications of these prediction models in ballistic studies, it is important that they realistically represent the relationship between striking and residual velocity and provide adequate estimates of the critical velocity V_c. Various applications of the penetration models mentioned above have demonstrated, however, that in many cases

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they do not sufficiently describe this relationship.

In this report we propose a procedure based on the versatile three-parameter Weibull distribution function for estimating the relationship between the residual and striking velocities of a projectile from test data. This model has many shapes, as illustrated in the next section, which should make it useful for fitting ballistic data over a wide range of firing conditions for various types of projectiles. In Section 3 we discuss nonlinear estimation procedures for fitting this model to test data. These procedures utilize striking velocities with zero residuals to help estimate the three unknown parameters, including the critical velocity. In Appendix A we list a computer program for estimating these parameters and illustrate its use by fitting the Weibull model to several sets of penetration data. In Section 4 we compare the Weibull and hyperbolic models on eight sets of penetration data.

2. THE WEIBULL MODEL

Consider a continuous functional relationship $V_r = G(V_s)$, between striking velocity and residual velocity, with other impact conditions fixed. We assume that the function G describing this relationship satisfies the following conditions:

(i) $G(V_s) = 0$ $\quad\quad\quad 0 \le V_s \le V_c$

(ii) $G'(V_s) > 0$ $\quad\quad\quad V_s > V_c$

(iii) $V_s > G(V_s)$ $\quad\quad\quad V_s > V_c$

(iv) $V_s - G(V_s) \to 0$ $\quad\quad V_s \to \infty$.

In practice, we are generally concerned with determining G over a finite range of V_s before any significant fragmentation of the projectile occurs. However, in the formulation of the problem, we assume the asymptotic property of condition (iv).

In the present paper we model the functional relationship as

$$G(V_s) = \begin{cases} 0 & V_s \le V_c \\ \\ V_s[1-e^{-\lambda(V_s-V_c)^\beta}] & V_s > V_c \end{cases} \quad\quad (2.1)$$

where $\lambda > 0$, $\beta > 0$ and $V_c > 0$. Observe that G satisfies conditions (i) - (iv) and has (for $V_s \ge V_c$) the form $G(x) = xF(x)$ where

$F(x) = 1-e^{-\lambda(x-\eta)^\beta}$, $x > \eta$, is the three-parameter Weibull distribution function with scale parameter λ, shape parameter β and location parameter η.

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In Figure 1 we show some of the many shapes the function G(x) can assume for various values of λ and β when $\eta = 100$. Condition (iv) implies, of course, that the line $y = x$ is an asymptote of G(x). However, the second derivative G"(x) is not necessarily less than zero for all x. Hence, as shown in Figure 1d, G(x) may actually move away from the line $y = x$ over a finite range of x, for certain values of λ and β. However, the sign of G"(x) will eventually change and G(x) will approach the line $y = x$ asymptotically. This property of G(x) is one of the characteristics which makes it a versatile model for fitting penetration data.

3. ESTIMATION PROCEDURES

The three unknown parameters λ, β and V_c in the model given by (2.1) can be estimated by the use of a nonlinear programming algorithm. A program, given in Appendix A, utilizing Marquardt's method [5] for nonlinear least squares has been developed by AMSAA for this application. Estimates of λ, β and V_c are determined as values $\hat{\lambda}$, $\hat{\beta}$ and $\hat{V}_c$, respectively, which minimize the root mean square error.

$$\text{ERMS} = \left(\frac{1}{N} \sum [V_r - G(V_s)]^2 \right)^{1/2},$$

where the summation is taken over all N pairs (V_s, V_r) such that $V_r > 0$.

The estimates $\hat{\lambda}$ and $\hat{\beta}$ are determined by this procedure with the constraint that they be greater than zero. Furthermore, to utilize the information associated with observed residual velocities that are zero, we perform this optimization with the additional constraint that $a \leq V_c < b$, where a and b are inputs to the program.

Marquardt's algorithm, which is an unconstrained optimization technique, has been modified to accommodate the constraints on λ, β and V_c. In most cases the algorithm converges to the solution within very few steps. If satisfactory results are not obtained by use of this method, one may elect to use another scheme such as those programmed by Wortman [6].

Regardless of the method employed to determine optimum values of the parameters, preliminary estimates $\lambda°$, $\beta°$, and $V_c°$ must be established. Discretion should be exercised in order to determine the interval [a,b) in which V_c is constrained to lie. It is often feasible to choose a to be

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Figure 1. Plots of $Y = X \left[1 - e^{-\lambda(X-\eta)^\beta} \right]$

(a) $\lambda = .1$, $\eta = 100$, $\beta = 2$

(b) $\lambda = .1$, $\eta = 100$, $\beta = .5$

(c) $\lambda = .3$, $\eta = 100$, $\beta = .5$

(d) $\lambda = .1$, $\eta = 100$, $\beta = .3$

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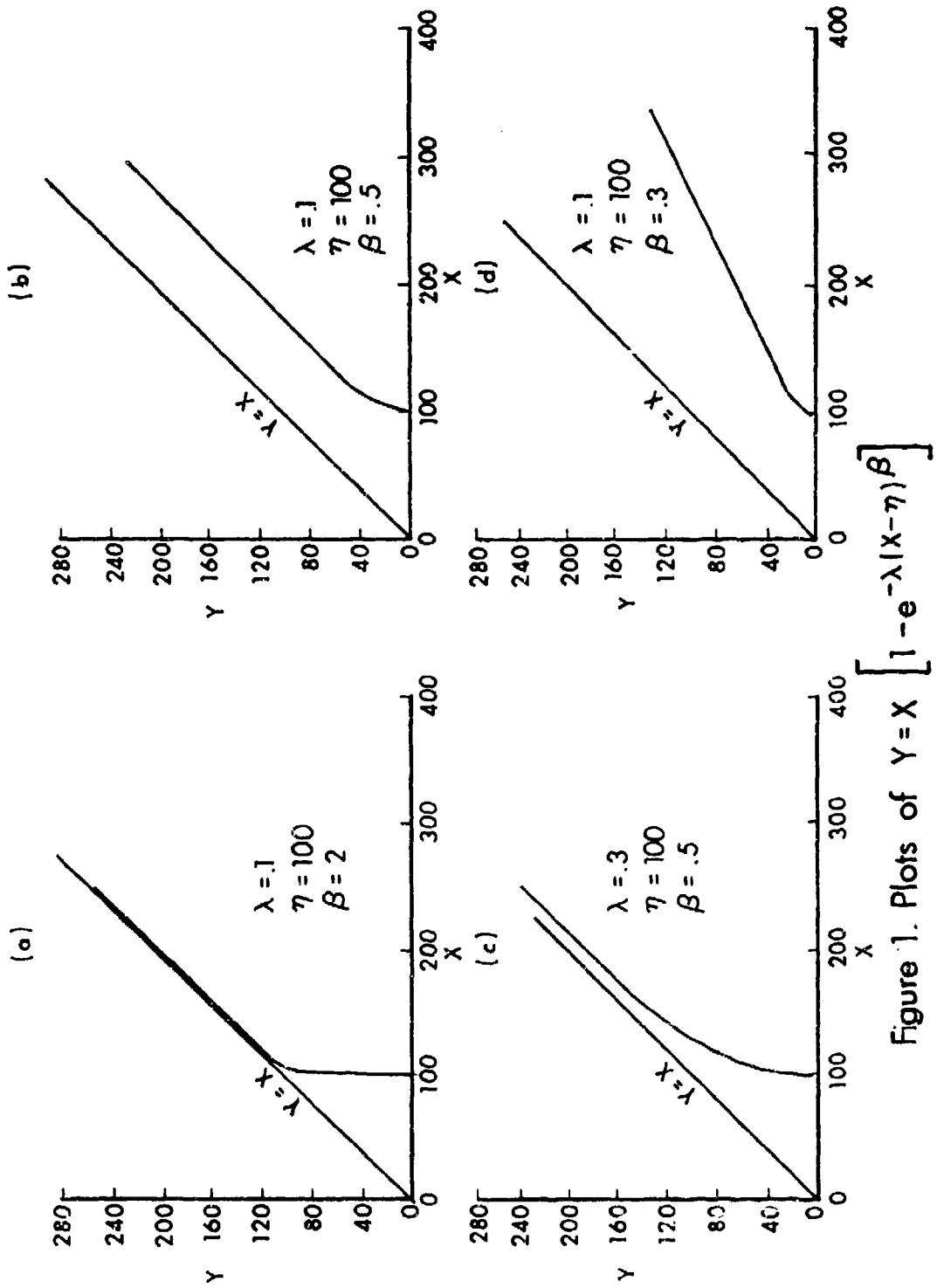
the highest striking velocity for which the projectile did not perforate
the target and b chosen to equal the lowest striking velocity for which
perforation was achieved. Once this interval has been determined an
initial estimate V_c^o may be arbitrarily chosen within the interval.

With this choice of V_c^o the equation (2.1) can be linearized to yield

$$\ln \ln \left(\frac{V_s}{V_s - V_r} \right) = \ln(\lambda) + \beta\ln(V_s - V_c).$$

In most cases initial estimates λ^o and β^o within the feasible region may
be then obtained by using linear regression.

In Appendix A we illustrate the application of these procedures
and the versatility of the model using several sets of penetration data.

4. COMPARISON OF WEIBULL AND HYPERBOLIC MODELS

In a recent study [3] published by the USA Ballistic Research Laboratories
the hyperbolic model $V_r^2 = AV_s^2 + B$ was used to analyze the residual
velocity of right circular cylinders after perforating doron body armor
material. The cylinders considered in this study were made from 01 Tool
Steel and heat treated to a hardness of R_c 29 $\pm$ 2. The hyperbolic model was
fitted to eight sets of penetration data from 2, 4, 16 and 64 grain cylinders
at 0° and 45° obliquity.

We fitted the Weibull model to the eight sets of data considered
in the above report and compared the results to those obtained from
the hyperbolic model. These comparisons, with the fitted curves, are
given in Tables 1-8 and Figures 2-9, respectively.

In four cases the ERMS's were slightly different for the two models
(the hyperbolic ERMS's being lower in three of these cases), while in
the other four cases the ERMS's for the Weibull model were significantly
lower. Also, note that the hyperbolic model, in several cases estimated
V_c lower than what one would expect based on the data. For example,
consider the case of 2 grain steel at 45 degrees obliquity. The hyper-
bolic model estimated V_c at 530 m/s, but the experimental data had non-
perforations for striking velocities as high as 629 m/s and perforations
for striking velocities only as low as 605 m/s. The Weibull model esti-
mate of V_c in this case is 607 m/s.

5. CONCLUSIONS

The Weibull model provides a versatile form which may be used to
represent the relationship between the striking velocity and the resid-
ual velocity of a projectile fired into some target material. This
model compares favorably with others proposed for this application.

In particular, it provides more satisfactory estimates of the critical velocity.

To date, this model has been used only to fit individual sets of data for which all conditions other than the striking velocity are held constant. No attempt has been made to physically interpret the values of the parameter estimates or to interpolate between test conditions. A model with those capabilities is needed for use in vulnerability and effectiveness models.

2 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 0 DEGREES OBLIQUITY

HYPERBOLIC MODEL $\quad VR^2 = -153388.9 + .5474 \, VS^2$

$$CRITICAL \; VELOCITY = 529.4 \; M/S$$
$$ERROR-RMS = 24.1 \; M/S$$

WEIBULL MODEL $\quad VR/VS = 1-EXP(- .0986(VS- 545.7)^{.36929})$

$$CRITICAL \; VELOCITY = 545.7 \; M/S$$
$$ERROR-RMS = 28.0 \; M/S$$

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
492.0	.0	524.0	.0
533.0	.0	537.0	.0
538.0	.0	550.0	.0
554.0	.0	547.0	76.0
551.0	80.0	555.0	98.0
561.0	63.0	607.0	181.0
655.0	259.0	671.0	309.0
713.0	374.0	794.0	455.0
888.0	528.0	960.0	594.0
986.0	636.0	1031.0	654.0
1068.0	685.0	1135.0	750.0
1176.0	789.0	1250.0	836.0
1342.0	902.0	1412.0	984.0
1461.0	1008.0	1534.0	1048.0
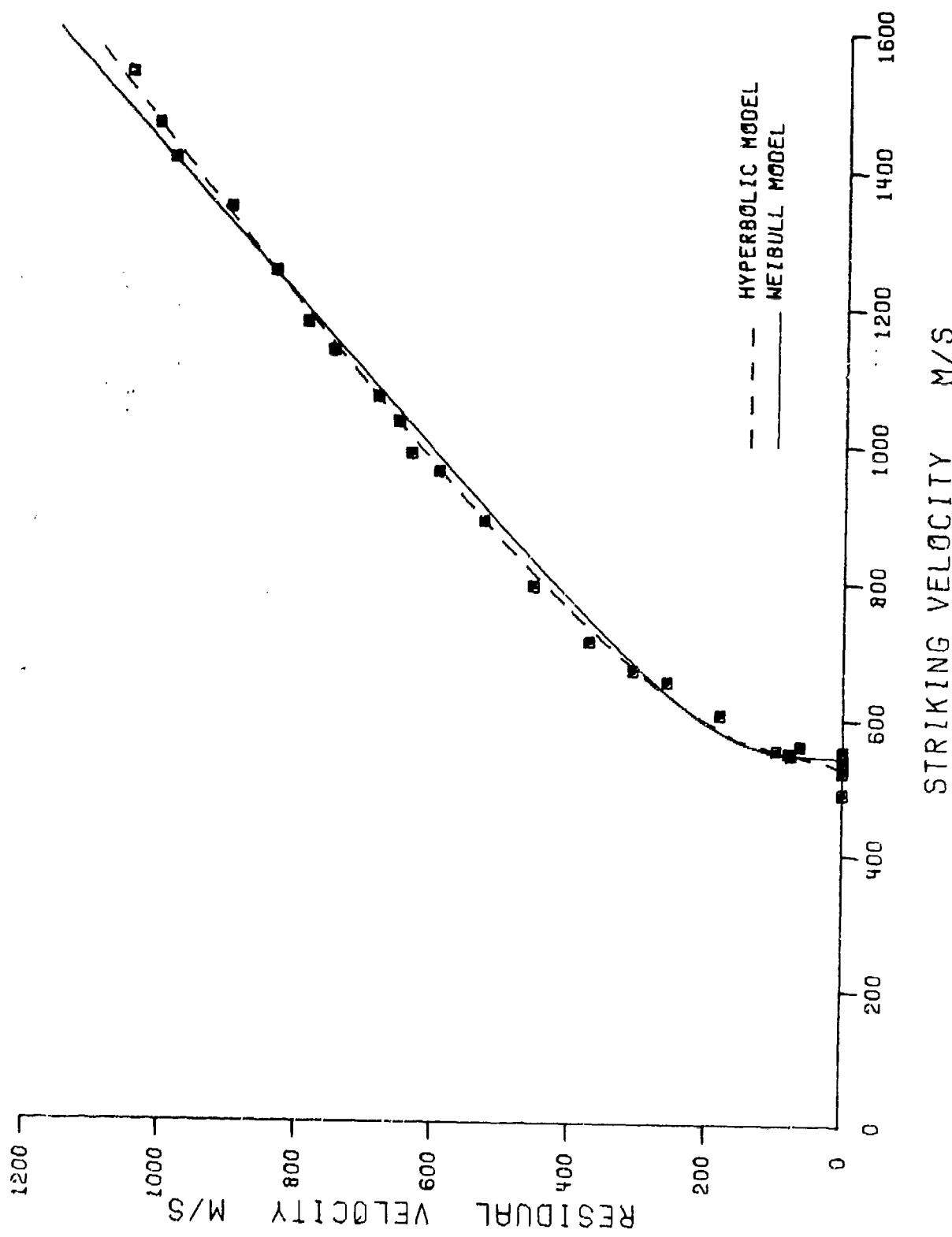
STRIKING VELOCITY M/S

2 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 0 DEGREES OBLIQUITY

FIGURE 2

— — — HYPERBOLIC MODEL

———— WEIBULL MODEL

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TABLE 2

4 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 0 DEGREES OBLIQUITY

HYPERBOLIC MODEL $\qquad VR^2 = -119248.1 + .6556 \ VS^2$

$$CRITICAL \ VELOCITY = 426.5 \ M/S$$
$$ERROR-RMS = 29.9 \ M/S$$

WEIBULL MODEL $\qquad VR/VS = 1-EXP(- .0777(VS- 421.2)^{.44043})$

$$CRITICAL \ VELOCITY = 421.2 \ M/S$$
$$ERROR-RMS = 34.7 \ M/S$$

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
281.0	.0	308.0	.0
402.0	.0	423.0	.0
431.0	.0	431.0	.0
441.0	.0	446.0	.0
447.0	.0	424.0	75.0
436.0	125.0	451.0	87.0
465.0	44.0	466.0	141.0
478.0	145.0	486.0	110.0
509.0	199.0	537.0	242.0
585.0	316.0	597.0	320.0
607.0	336.0	622.0	360.0
657.0	397.0	662.0	415.0
689.0	443.0	773.0	535.0
792.0	555.0	807.0	569.0
817.0	591.0	879.0	638.0
903.0	663.0	975.0	721.0
996.0	736.0	1027.0	768.0
1037.0	776.0	1067.0	794.0
1116.0	833.0	1143.0	851.0
1191.0	894.0	1247.0	945.0
1283.0	974.0	1290.0	969.0
1297.0	1002.0	1306.0	990.0
1341.0	1031.0		

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STRIKING VELOCITY M/S

RESIDUAL VELOCITY M/S

--- HYPERBOLIC MODEL
—— WEIBULL MODEL
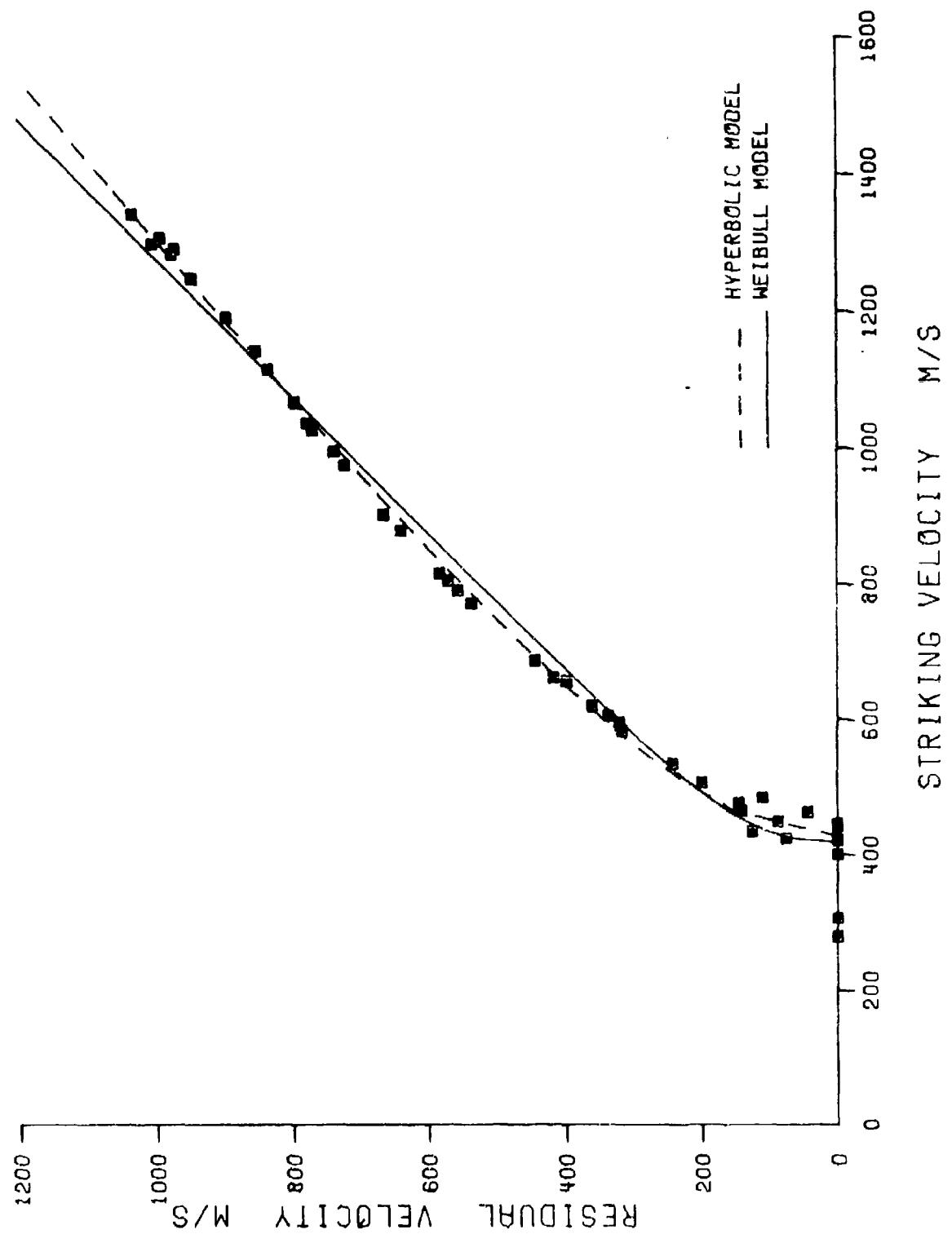
4 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD BORON AT 0 DEGREES OBLIQUITY
FIGURE 3

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TABLE 3

16 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 0 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR^2 = -58815.3 + .7782 \ VS^2$

CRITICAL VELOCITY = 274.9 M/S
ERROR-RMS =   15.3 M/S

WEIBULL MODEL $VR/VS = 1-EXP(- .2440(VS- 309.1)^{.31345})$

CRITICAL VELOCITY = 309.1 M/S
ERROR-RMS =   16.1 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
282.0	.0	284.0	.0
287.0	.0	312.0	96.0
322.0	120.0	349.0	170.0
397.0	236.0	477.0	333.0
537.0	404.0	615.0	482.0
693.0	562.0	773.0	656.0
874.0	747.0	881.0	756.0
953.0	808.0	1037.0	882.0
1109.0	940.0	1195.0	1024.0
1251.0	1069.0		

STRIKING VELOCITY M/S

16 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT O DEGREES OBLIQUITY
FIGURE 4

RESIDUAL VELOCITY M/S

— — — HYPERBOLIC MODEL
——— WEIBULL MODEL

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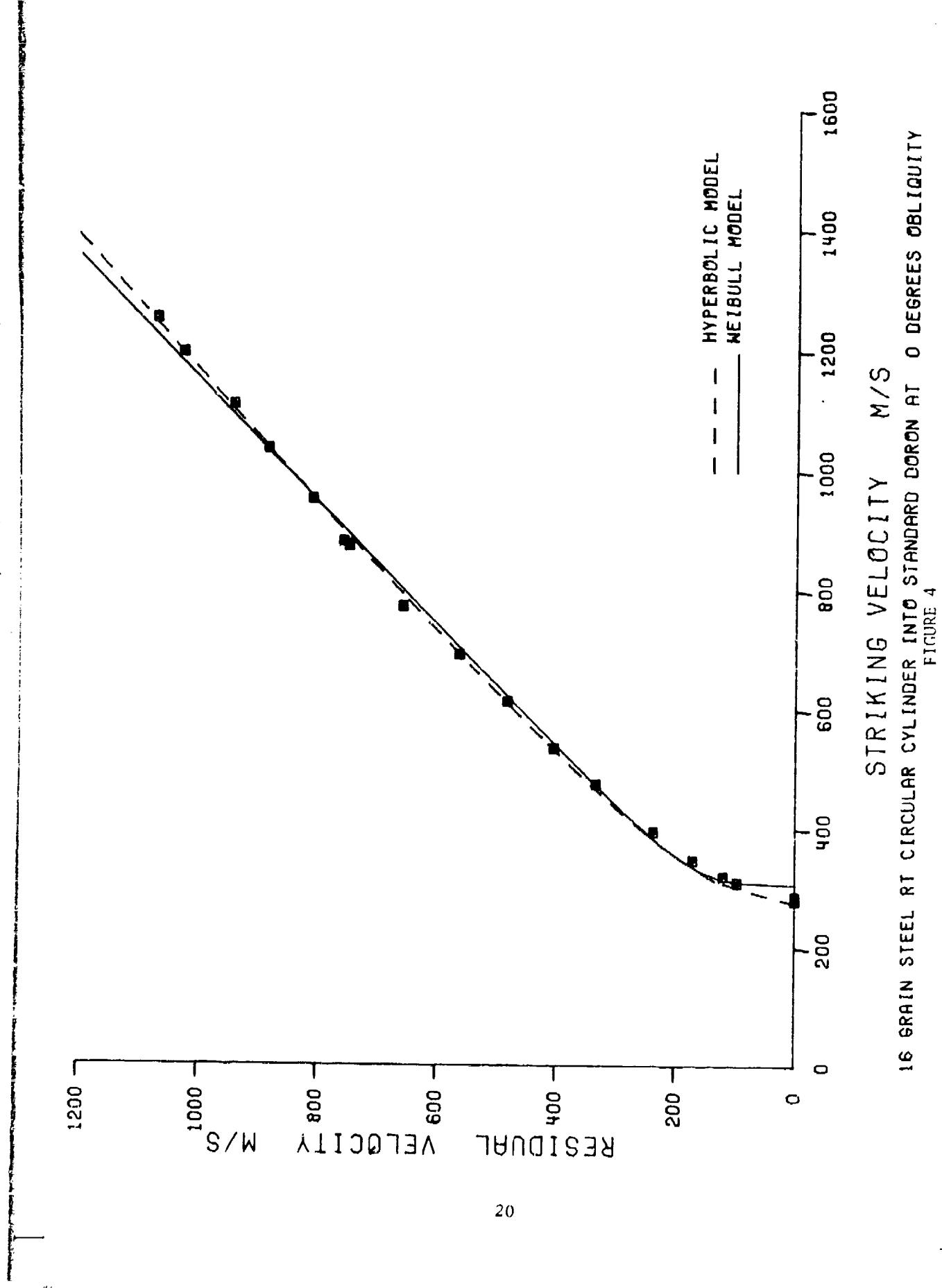
TABLE 4

64 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 0 DEGREES OBLIQUITY

HYPERBOLIC MODEL $\quad VR^2 = -31554.4 + .8533 \, VS^2$

$$\text{CRITICAL VELOCITY} = 192.3 \text{ M/S}$$
$$\text{ERROR-RMS} = 17.6 \text{ M/S}$$

WEIBULL MODEL $\quad VR/VS = 1-EXP(- .4322(VS- 245.0)^{.26232})$

$$\text{CRITICAL VELOCITY} = 245.0 \text{ M/S}$$
$$\text{ERROR-RMS} = 16.6 \text{ M/S}$$

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
193.0	.0	208.0	.0
223.0	.0	228.0	.0
249.0	121.0	250.0	118.0
277.0	163.0	277.0	200.0
278.0	160.0	300.0	200.0
308.0	240.0	309.0	203.0
314.0	260.0	326.0	222.0
384.0	293.0	461.0	418.0
551.0	479.0	554.0	488.0
633.0	558.0	635.0	543.0
635.0	560.0	707.0	638.0
809.0	728.0	927.0	843.0
989.0	899.0	1153.0	1043.0
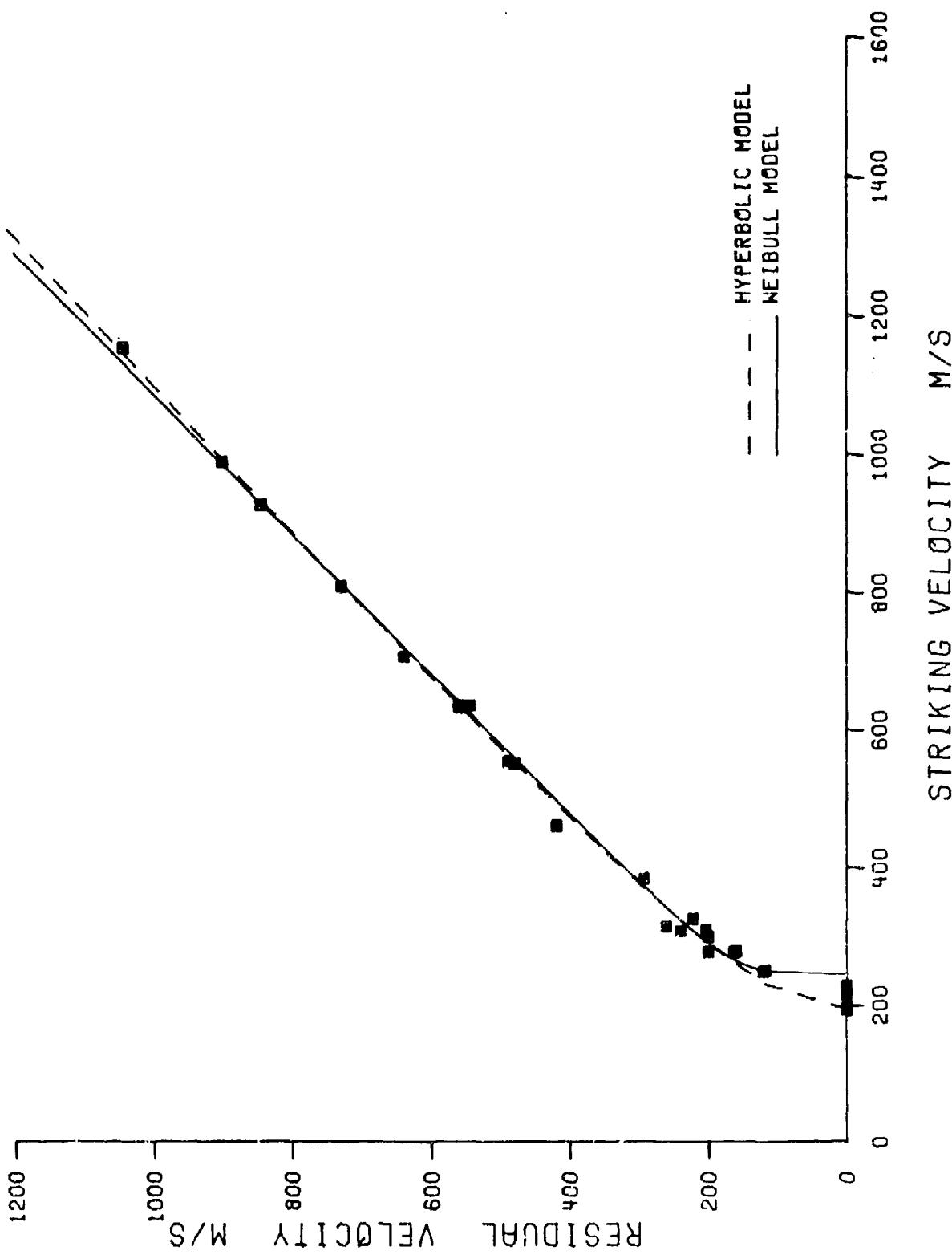
STRIKING VELOCITY M/S

RESIDUAL VELOCITY M/S

– – – HYPERBOLIC MODEL
——— WEIBULL MODEL

64 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 0 DEGREES OBLIQUITY

FIGURE 5

TABLE 5

2 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY

HYPERBOLIC MODEL $\quad VR^2 = -114876.9 + .4091 \ VS^2$

$$CRITICAL \ VELOCITY = 529.9 \ M/S$$
$$ERROR-RMS = 51.3 \ M/S$$

WEIBULL MODEL $\quad VR/VS = 1-EXP(- .1656(VS- 606.9)^{.25822})$

$$CRITICAL \ VELOCITY = 606.9 \ M/S$$
$$ERROR-RMS = 40.7 \ M/S$$

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
544.0	.0	561.0	.0
577.0	.0	587.0	.0
587.0	.0	600.0	.0
601.0	.0	609.0	.0
615.0	.0	617.0	.0
621.0	.0	629.0	.0
605.0	72.0	607.0	51.0
620.0	140.0	628.0	170.0
632.0	194.0	635.0	235.0
636.0	183.0	657.0	192.0
671.0	208.0	678.0	184.0
680.0	217.0	714.0	361.0
740.0	378.0	781.0	418.0
789.0	406.0	798.0	417.0
855.0	413.0	863.0	529.0
877.0	421.0	901.0	525.0
931.0	497.0	960.0	526.0
1018.0	574.0	1067.0	619.0
1078.0	595.0	1172.0	644.0
1209.0	696.0	1221.0	679.0
1235.0	698.0	1341.0	780.0
1414.0	839.0	1457.0	867.0
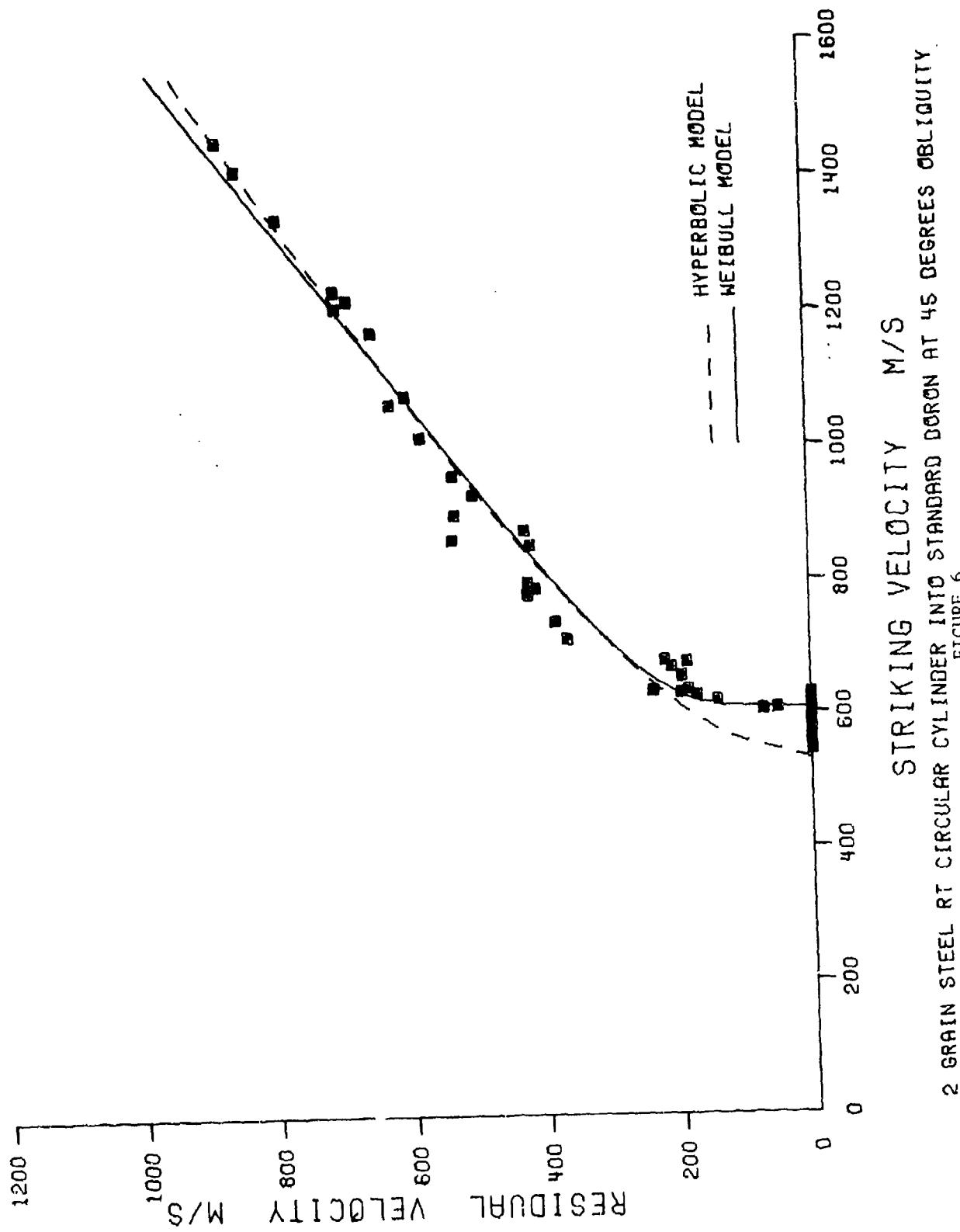
STRIKING VELOCITY M/S

RESIDUAL VELOCITY M/S

- - - HYPERBOLIC MODEL
——— WEIBULL MODEL

2 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY
FIGURE 6

TABLE 6

4 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY

HYPERBOLIC MODEL $\quad VR^2 = -72019.2 + .5039\ VS^2$

$$CRITICAL\ VELOCITY = 378.1\ M/S$$
$$ERROR-RMS = 55.2\ M/S$$

WEIBULL MODEL $\quad VR/VS = 1-EXP(- .3015(VS- 494.3)^{.20376})$

$$CRITICAL\ VELOCITY = 494.3\ M/S$$
$$ERROR-RMS = 40.9\ M/S$$

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
423.0	.0	437.0	.0
454.0	.0	459.0	.0
467.0	.0	474.0	.0
475.0	.0	480.0	.0
481.0	.0	483.0	.0
483.0	96.0	495.0	134.0
497.0	123.0	505.0	148.0
536.0	204.0	575.0	275.0
629.0	366.0	671.0	411.0
688.0	428.0	717.0	459.0
799.0	544.0	840.0	585.0
919.0	646.0	1002.0	694.0
1171.0	787.0	1285.0	866.0
1299.0	872.0	1348.0	893.0
1348.0	912.0		

STRIKING VELOCITY M/S
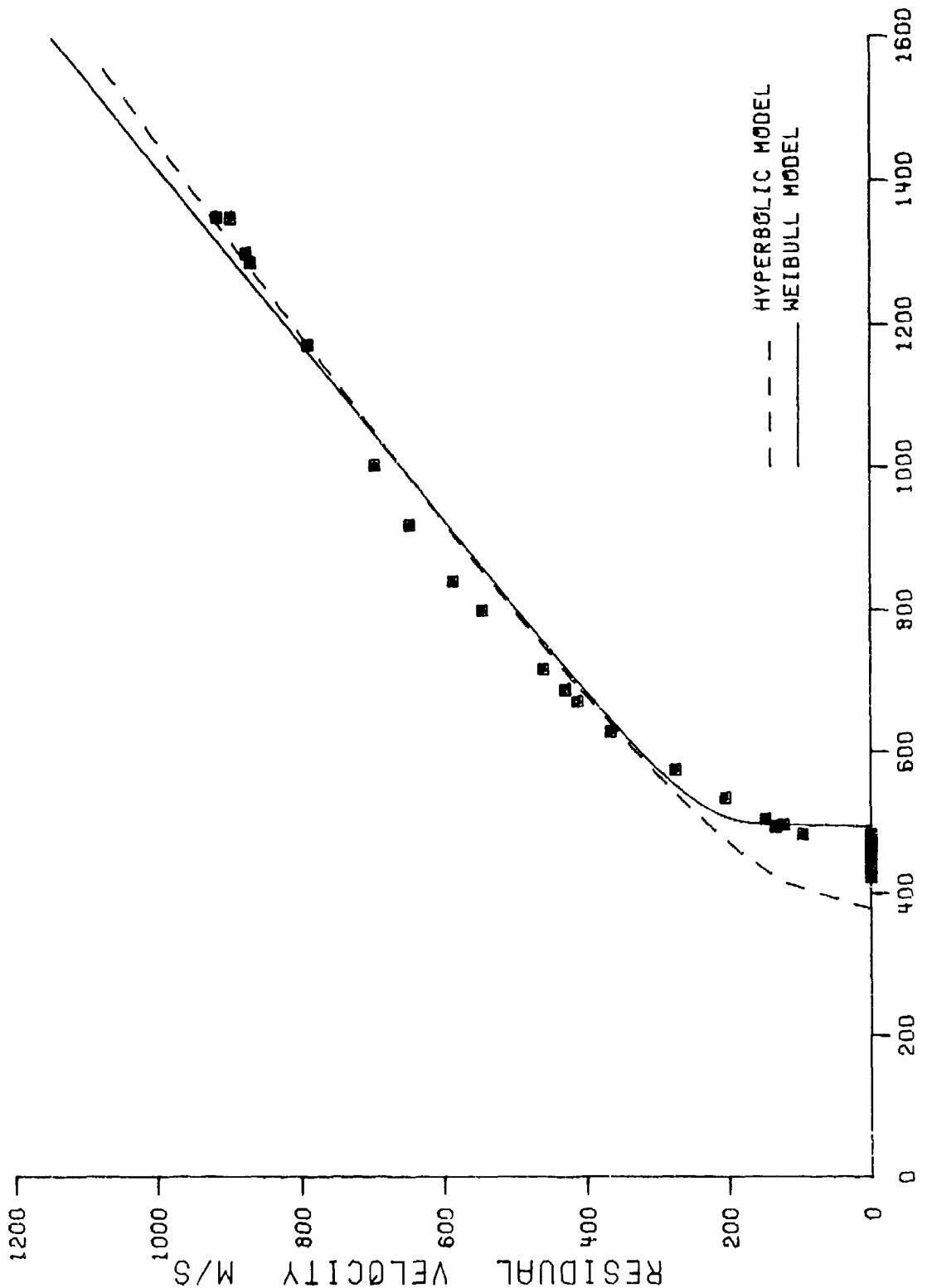
4 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY

FIGURE 7

26

TABLE 7

16 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 45 DEGREES OBLIQUITY

HYPERBOLIC MODEL $VR^2 = -50639.3 + .6889\ VS^2$

CRITICAL VELOCITY =   271.1 M/S
ERROR-RMS =   35.9 M/S

WEIBULL MODEL $VR/VS = 1-EXP(- .2515(VS- 323.3)^{.28592})$

CRITICAL VELOCITY =   323.3 M/S
ERROR-RMS =   26.6 M/S

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
302.0	.0	304.0	.0
304.0	.0	313.0	.0
324.0	.0	315.0	38.0
324.0	78.0	327.0	90.0
335.0	96.0	335.0	.00.0
348.0	111.0	370.0	169.0
408.0	234.0	433.0	278.0
486.0	367.0	508.0	361.0
545.0	393.0	567.0	421.0
587.0	441.0	665.0	518.0
699.0	549.0	797.0	639.0
859.0	693.0	905.0	734.0
955.0	772.0	964.0	778.0
979.0	791.0	1005.0	811.0
1026.0	834.0	1138.0	919.0
1159.0	935.0	1207.0	944.0
1227.0	984.0	1234.0	990.0

STRIKING VELOCITY M/S

— — — HYPERBOLIC MODEL
———— WEIBULL MODEL
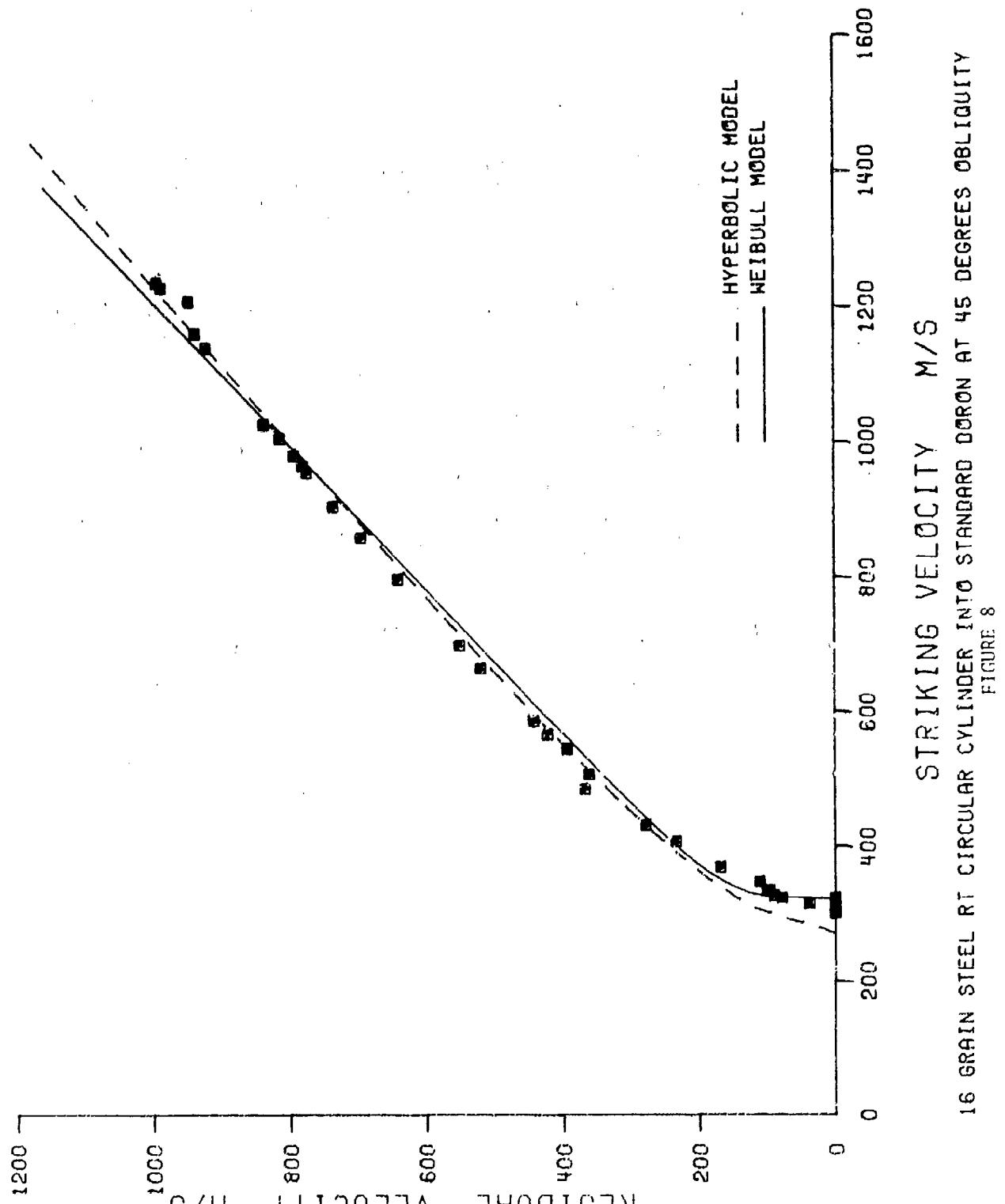
16 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY
FIGURE 8

RESIDUAL VELOCITY M/S

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TABLE 8

64 GRAIN STEEL RT CIRCULAR CYLINDER INTO
STANDARD DORON AT 45 DEGREES OBLIQUITY

HYPERBOLIC MODEL $\quad VR^2 = -15795.6 + .7950\ VS^2$

$$\text{CRITICAL VELOCITY} = 141.0\ M/S$$
$$\text{ERROR-RMS} = 38.2\ M/S$$

WEIBULL MODEL $\quad VR/VS = 1-EXP(- .6628(VS- 240.0)^{.18142})$

$$\text{CRITICAL VELOCITY} = 240.0\ M/S$$
$$\text{ERROR-RMS} = 31.8\ M/S$$

EXPERIMENTAL DATA

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S
211.0	.0	212.0	.0
213.0	.0	215.0	.0
225.0	77.0	233.0	64.0
243.0	120.0	244.0	116.0
271.0	162.0	275.0	165.0
334.0	290.0	379.0	290.0
383.0	362.0	451.0	412.0
535.0	496.0	626.0	576.0
674.0	588.0	733.0	644.0
790.0	671.0	792.0	697.0
852.0	755.0	904.0	803.0
967.0	859.0	1013.0	902.0
1067.0	948.0	1114.0	987.0
1159.0	1028.0	1203.0	1042.0
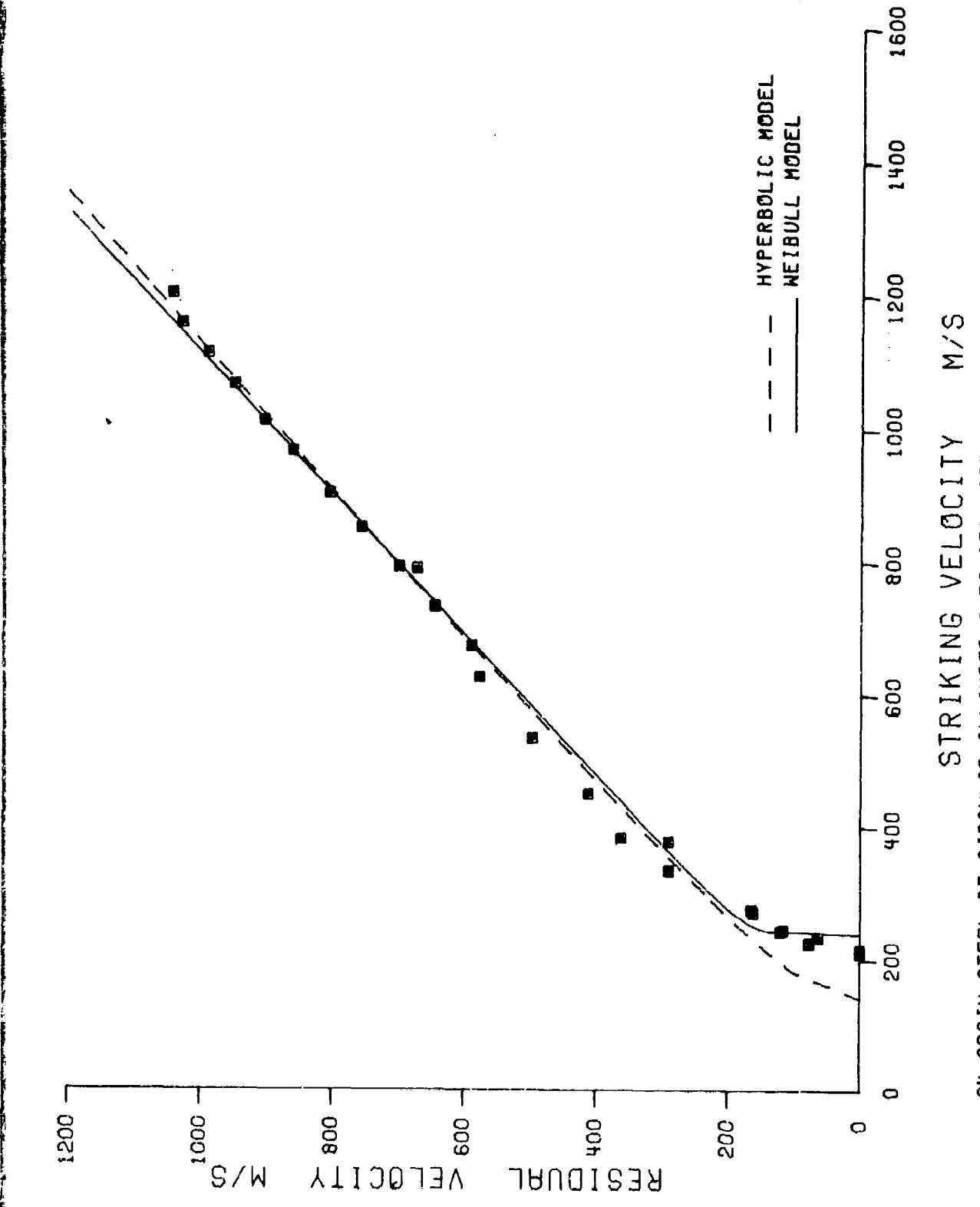
STRIKING VELOCITY M/S

64 GRAIN STEEL RT CIRCULAR CYLINDER INTO STANDARD DORON AT 45 DEGREES OBLIQUITY

FIGURE 9

— — — HYPERBOLIC MODEL

——— WEIBULL MODEL

RESIDUAL VELOCITY M/S

REFERENCES

1. W. Bruchey, Jr., "A Comparison of the Residual Velocities of Various Fragment Simulating Projectiles and Actual Munitions Fragments After Penetrating Nylon Body Armor Material (U)," Ballistic Research Laboratories Interim Memorandum Report No. 22, Nov 1971 (CONFIDENTIAL) (no longer available).

2. W. Johnson, C. Collins and F. Kindred, "A Mathematical Model for Predicting Residual Velocities of Fragments After Perforating Helmets and Body Armor (U)," Ballistic Research Laboratories Technical Note No. 1705, October 1968. (CONFIDENTIAL) (AD# 394512)

3. W. Kokinakis, and F. H. Essig, "Penetration of Doron Body Armor Material By A Right Circular Cylinder Fragment Simulator," Ballistic Research Laboratories Memorandum Report No. 2445, March 1975.

4. P. G. Morfogenis, "A Learning Curve Type Equation Predicting Residual Velocity (U)," Ballistic Research Laboratories Memorandum Report No. 2477, April 1975.

5. D. W. Marquardt, "An Algorithm for Least Square Estimation of Nonlinear Parameter," J. SIAM, 2, 1963, pp. 431-441.

6. J. D. Wortman, "NLPROG (A Set of FORTRAN Programs to Find The Minimum of a Constrained Function)" Ballistic Research Laboratories Memorandum Report No. 1958, January 1969.

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Appendix A

A FORTRAN computer program has been prepared to provide estimates of the parameters for the Weibull model described in this report. The program was written for use on the Ballistic Research Laboratories computers, BRLESC I and II. The program utilizes 16K memory locations. The CALCOMP plotter package is used to provide plots of the data and fitted curves.

The required input is as follows:

	Columns	
Card 1	1-79	Alphanumeric string used for title
	80	The symbol ">"
Card 2	1-10	Minimum value for critical velocity
	11-20	Maximum value for critical velocity
Card 3	1-10	Striking velocity
	11-20	Residual velocity

Repeat Card 3 as necessary to complete the data set. Follow the last Card 3 with a blank card.

After execution of the computation for a data set the program returns to read card 1 for another set.

A listing of the source program is provided on the following pages. To provide several samples of output and also to demonstrate the flexibility of the Weibull model the output from several data sets is also included. These data were obtained from firings of a 90 percent tungsten spheriod weighing approximately 7 grains. The firing conditions include two target materials, two thicknesses, and two angles of obliquity.

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```
      PROGRAM WBLRV                                                    WBLRV 1
      COMMON /HPX/ HMAX, HMIN                                          WBLRV 2
      DIMENSION VS(100), VR(100), Z(100,2), A(2,3), C(3), R(100),      WBLRV 3
     1  AF(100), SIG(3), T(3), ITL(8), F(100), TX(3), TY(3)            WBLRV 4
      DIMENSION XZ(20), YZ(20)                                         WBLRV 5
      EXTERNAL VREM                                                    WBLRV 6
      DATA TX(1), TX(2), TY(1), TY(2) /10HSTRIKING V, 8HELOCITY , 10HRESWBLRV 7D
     1IDUAL  , 9HVELOCITY / , TX(3), TY(3) /4HM/S>, 4HM/S>/            WBLRV 8D
      INN=1                                                            WBLRV 9
    1 READ (5,11) (ITL(I),I=1,8)                                       WBLRV10R
      WRITE (6,13) (ITL(I),I=1,8)                                      WBLRV11W
      READ (5,16) HMIN,HMAX                                           WBLRV12R
      WRITE (6,18) HMIN,HMAX                                          WBLRV13W
      M=1                                                              WBLRV14
      NZ=C                                                             WBLRV15
    2 READ (5,12) VS(M),VR(M)                                          WBLRV16R
      IF (VS(M).LE.0.) GO TO 4                                         WBLRV17
      IF (VR(M).LE.0.) GO TO 3                                         WBLRV18
      IF (VS(M).LE.HMIN) GO TO 3                                       WBLRV19
      M=M+1                                                            WBLRV20
      GO TO 2                                                          WBLRV21
    3 NZ=NZ+1                                                          WBLRV22
      XZ(NZ)=VS(M)                                                     WBLRV23
      YZ(NZ)=VR(M)                                                     WBLRV24
      GO TO 2                                                          WBLRV25
    4 M=M-1                                                            WBLRV26
      IF (NZ.GT.C.) CALL SORTXY (XZ,YZ,NZ)                             WBLRV27
      CALL SORTXY (VS,VR,M)                                            WBLRV28
      H=HMIN                                                           WBLRV29
      DO 5 K=1,M                                                       WBLRV30
      F(K)=ALOG(ALOG(VS(K)/(VS(K)-VR(K))))                             WBLRV31
      Z(K,1)=1.                                                        WBLRV32
      Z(K,2)=ALOG(VS(K)-H)                                             WBLRV33
    5 CONTINUE                                                         WBLRV34
      IF (M.GT.2) GO TO 6                                              WBLRV35
      WRITE (6,17) M                                                   WBLRV36W
      GO TO 1                                                          WBLRV37
    6 CALL GENLSQ (Z,100,F,M,A,2,2,C,R,AF,ERMS,SIG,T,DET,1)            WBLRV38
      DO 7 I=1,M                                                       WBLRV39
      F(I)=VR(I)                                                       WBLRV40
      Z(I,1)=VS(I)                                                     WBLRV41
    7 Z(I,2)=VR(I)                                                     WBLRV42
      C(3)=C(2)                                                        WBLRV43
      C(2)=EXP(C(1))                                                   WBLRV44
      C(1)=H                                                           WBLRV45
      WRITE (6,19) (C(I),I=1,3)                                        WBLRV46W
      CALL MRQLS (VREM,F,Z,C,2,M,100,3,ERMS,SIG,R,AF,.1E-5,.1E-3)      WBLRV47
      B=C(3)                                                           WBLRV48
      XL=C(2)                                                          WBLRV49
      H=C(1)                                                           WBLRV50
      WRITE (6,20) H,XL,B,ERMS                                         WBLRV51W
      WRITE (6,21)                                                     WBLRV52W
      IF (NZ.GT.0) WRITE (6,15) (XZ(I),YZ(I),I=1,NZ)                   WBLRV53W
      DO 8 I=1,M                                                       WBLRV54
    8 WRITE (6,14) VS(I),VR(I),AF(I),R(I)                              WBLRV55W
      YMAX=1500.                                                       WBLRV56
      YS=YMAX/6.                                                       WBLRV57
      XOR=0.                                                           WBLRV58
      XMAX=XOR+8.*YS                                                   WBLRV59
      CALL PLTVG (VS,VR,M,XOR,0.,YS,YS,TX,TY,2,5,INN,ITL)              WBLRV60
      IF (NZ.GT.0) CALL PLTCCD (2,5,XZ(1),YZ(1),NZ)                    WBLRV61
      VS(1)=H                                                          WBLRV62
      VR(1)=0.                                                         WBLRV63
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```
      DX=.C25*YS                                                     WBLRV64
      I=1                                                            WBLRV65
    9 VSAV=VR(I)+DX                                                  WBLRV66
      VST=VS(I)+DX                                                   WBLRV67
      IF (VST.GT.XMAX) GO TO 10                                      WBLRV68
      I=I+1                                                          WBLRV69
      VS(I)=VST                                                      WBLRV70
      VR(I)=VS(I)*(1.-EXP(-XL*(VS(I)-H)**B))                         WBLRV71
      VSAV=VR(I)                                                     WBLRV72
      IF (I.GT.40) DX=.25*YS                                         WBLRV73
      IF (VSAV.LT.YMAX) GO TO 9                                      WBLRV74
   1C CALL PLTCCO (1,,VS(1),VR(1),I,0,XOR,XMAX,0.,YMAX)              WBLRV75
      IAN=C                                                          WBLRV76
      GO TC 1                                                        WBLRV77
C                                                                    WBLRV78
   11 FORMAT (8A1C)                                                  WBLRV79
   12 FCRMAT (2F1C.0)                                                WBLRV80
   13 FORMAT (1H1,30X,4A10/30X,3A10,A9/)                             WBLRV81
   14 FCRMAT (19X,4F14.1)                                            WBLRV82
   15 FCRMAT (19X,2F14.1)                                            WBLRV83
   16 FORMAT (2F1C.5)                                                WBLRV84
   17 FCRMAT (5HOCALY,I3,25H DATA PCINTS--FIT OMITTED)               WBLRV85
   18 FORMAT (2CX,53HCRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FRWBLRV86
     1CM,F7.1/2CX,3HTC ,F7.1,5H M/S.//)                             WBLRV87
   19 FCRMAT (2CX,13HWEIBULL MODEL,8X,6HVC M/S,6X,24HLAMBCA    BETA   WBLRV88
     1 ERMS//2CX,17HINITIAL ESTIMATES,F10.1,3X,2F10.6/)             WBLRV89
   2C FCRMAT (2CX,15HFINAL ESTIMATES,F12.1,3X,2F10.6,F7.1///)        WBLRV90
   21 FCRMAT (26X,49HSTRIKING     RESIDUAL     APPROXIMATION     ERROR/2WBLRV91
     16X,22HVELCCITY        VELOCITY/18X,4(11X,3H(M/S)/)             WBLRV92
      END                                                           WBLRV93-
      SUBROUTINE VREM (B,X,F,P,IC,M,K)                              VREM  1
      CCMMCN /HMX/ HMAX,HMIN                                        VREM  2
      DIMENSION B(K), X(M), P(K)                                    VREM  3
      IF (B(1).GT.HMAX) B(1)=HMAX                                   VREM  4
      IF (B(1).LT.HMIN) B(1)=HMIN                                   VREM  5
      XP=X(1)-P(1)                                                  VREM  6
      IF (XP.GT.0.) GO TO 1                                         VREM  7
      F=C.                                                          VREM  8
      GC TC 2                                                       VREM  9
    1 XPL=ALOC(XP)                                                  VREM 10
      XPB=EXP(B(3)*XPL)                                             VREM 11
      EX=EXP(-B(2)*XPB)                                             VREM 12
      F=X(1)*(1.-EX)                                                VREM 13
    2 IF(IC.NE.C) RETURN                                            VREM 14
      IF(XP.GT.C)GC TO 3                                            VREM 15
      P(1)=0.                                                       VREM 16
      P(2)=C.                                                       VREM 17
      P(3)=C.                                                       VREM 18
      RETLRN                                                        VREM 19
    3 P(1)=-EX*X(1)*B(2)*B(3)*XPB/XP                                VREM 20
      P(2)=X(1)*XPB*EX                                              VREM 21
      P(3)=P(2)*B(2)*XPL                                            VREM 22
      RETLRN                                                        VREM 23
      END                                                           VREM 24
      SUBROUTINE MRQLS (FORM,Y,X,B,M,N,NMAX,K,ERMS,SE,R,F,TAU,EPS)   MRQLS 1
      DIMENSION Y(N), X(NMAX,M), B(K), SE(K), R(N), F(N)            MRQLS 2
      DIMENSION A(20,20), SA(20,20), P(20), V(10), G(20), SG(20)    MRQLS 3
      GNL=1C.                                                       MRQLS 4
      ICT=0                                                         MRQLS 5
      XL=.C1                                                        MRQLS 6
```

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  1 IC=C                                                        MRQLS 7
    STEP=1.                                                      MRQLS 8
    ICT=ICT+1                                                    MRQLS 9
    PHI=0.                                                       MRQLS10
    DO 2 I=1,20                                                  MRQLS11
    G(I)=0.                                                      MRQLS12
    DC 2 J=1,20                                                  MRQLS13
  2 A(I,J)=0.                                                    MRQLS14
    DO 5 I=1,N                                                   MRQLS15
    DC 3 J=1,M                                                   MRQLS16
  3 V(J)=X(I,J)                                                  MRQLS17
    CALL FORM (B,V,F(I),P,IC,M,K)                               MRQLS18
    R(I)=Y(I)-F(I)                                               MRQLS19
    PHI=PHI+R(I)**2                                             MRQLS20
    CC 4 J=1,K                                                   MRQLS21
    G(J)=G(J)+R(I)*P(J)                                         MRQLS22
    DO 4 L=J,K                                                   MRQLS23
  4 A(J,L)=A(J,L)+P(J)*P(L)                                     MRQLS24
  5 CONTINUE                                                     MRQLS25
    ERMS=SQRT(PHI/FLOAT(N))                                     MRQLS26
    DC 6 I=1,K                                                   MRQLS27
    SE(I)=SQRT(A(I,I))                                          MRQLS28
  6 G(I)=G(I)/SE(I)                                             MRQLS29
    DC 7 I=1,K                                                   MRQLS30
    DC 7 J=1,K                                                   MRQLS31
    A(I,J)=A(I,J)/(SE(I)*SE(J))                                 MRQLS32
    IF (J.GT.I) A(J,I)=A(I,J)                                   MRQLS33
                                                                MRQLS34
  7 CCNTINUE                                                     MRQLS35
    DO 8 I=1,K                                                   MRQLS36
    SG(I)=G(I)                                                   MRQLS37
    DO 8 J=1,K                                                   MRQLS38
  8 SA(I,J)=A(I,J)                                              MRQLS39
    IC=1                                                         MRQLS40
    XLM=XL/GNU                                                   MRQLS41
  9 DC 1C I=1,K                                                  MRQLS42
 10 A(I,I)=A(I,I)+XLM                                           MRQLS43
    CALL MATINV (A,K,G,20,1,DET)                               MRQLS44
    IF (DET.EQ.0.) GO TO 17                                    MRQLS45
 11 DO 12 I=1,K                                                  MRQLS46
 12 G(I)=B(I)+STEP*(G(I)/SE(I))                                MRQLS47
    PHIL=0.                                                     MRQLS48
    DC 14 I=1,N                                                  MRQLS49
    DO 13 J=1,M                                                  MRQLS50
 13 V(J)=X(I,J)                                                  MRQLS51
    CALL FORM (G,V,F(I),P,IC,M,K)                               MRQLS52
    R(I)=Y(I)-F(I)                                              MRQLS53
 14 PHIL=PHIL+R(I)**2                                           MRQLS54
    GO TO (15,19), IC                                           MRQLS55
 15 IF (PHIL-PHI) 16,16,17                                      MRQLS56
 16 XL=XLM                                                       MRQLS57
    GO TO 25                                                     MRQLS58
 17 DO 18 I=1,K                                                  MRQLS59
    G(I)=SG(I)                                                  MRQLS60
    DO 18 J=1,K                                                  MRQLS61
 18 A(I,J)=SA(I,J)                                              MRQLS62
    IC=2                                                         MRQLS63
    XLM=XL                                                       MRQLS64
    GO TC 9                                                      MRQLS65
 19 IF (PHIL-PHI) 25,25,20                                      MRQLS66
 20 IF (STEP.LT.1.) GO TO 24
```

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      IF (XL.GE.GNL*GNU) GO TO 22                                        MRQLS67
   21 XL=XL*GNU                                                          MRQLS68
      GO TO 17                                                           MRQLS69
   22 GNCRM2=0.                                                          MRQLS70
      DNCRM2=C.                                                          MRQLS71
      COT=C.                                                             MRQLS72
      DC 23 I=1,K                                                        MRQLS73
      GI=SG(I)*SE(J)                                                     MRQLS74
      DI=G(I)-B(I)                                                       MRQLS75
      DOT=DCT+GI*DI                                                      MRQLS76
      GNCRM2=GNCRM2+GI*GI                                                MRQLS77
   23 DNCRM2=DNCRM2+DI*DI                                                MRQLS78
      DOT=DOT/SCRT(GNCRM2*DNORM2)                                        MRQLS79
   24 IF (DOT.LE..7C71) GO TO 21                                         MRQLS80
      STEP=.5*STEP                                                       MRQLS81
      GC TO 11                                                           MRQLS82
   25 DC 26 I=1,K                                                        MRQLS83
      IF (ABS(G(I)-B(I))/(TAU+ABS(B(I))).GT.EPS) GO TO 29               MRQLS84
   26 CONTINUE                                                           MRQLS85
   27 ERRS=SQRT(PHIL/FLOAT(N))                                           MRQLS86
      DC 28 I=1,K                                                        MRQLS87
   28 B(I)=G(I)                                                          MRQLS88
      RETURN                                                             MRQLS89
   29 DC 30 I=1,K                                                        MRQLS90
   30 B(I)=G(I)                                                          MRQLS91
      IF (ICT.LT.5C) GO TO 1                                             MRQLS92
      GO TO 27                                                           MRQLS94
C                                                                        MRQLS95
   31 FCRMAT (1HC,18HFAILED TO CONVERGE)                                 MRQLS96
      END                                                                MRQLS97-
      SUBROUTINE PLTVG (X,Y,N,XOR,YOR,XS,YS,TX,TY,M,NS,INN,ITL)          PLTVG 1
      DIMENSION X(N), Y(N), TX(2), TY(2), B(5000), XC(2), YC(2), ITL(8)  PLTVG 2
      IF (INN.NE.1) GO TO 1                                              PLTVG 3
      CALL PLTCCR (12.,1,B(1),B(5COC))                                   PLTVG 4
      ICT=2                                                              PLTVG 5
    1 I=ICT+1                                                            PLTVG 6
      ICT=MOD(I,3)                                                       PLTVG 7
      IF (ICT.EC.C) CALL PLTCCP                                          PLTVG 8
      XB=1.                                                              PLTVG 9
      YB=1.+9.5*FLOAT(ICT)                                               PLTVG10
      CALL PLTCCS (XB,YB,0.,0.,1.,1.)                                    PLTVG11
      DC 3 I=1,2                                                         PLTVG12
      XP=1C.5*FLOAT(I-1)                                                 PLTVG13
      DC 2 J=1,2                                                         PLTVG14
      YP=8.*FLOAT(J-1)                                                   PLTVG15
      XC(1)=XP                                                           PLTVG16
      XC(2)=XP                                                           PLTVG17
      YC(1)=YP-.25                                                       PLTVG18
      YC(2)=YP+.25                                                       PLTVG19
      CALL PLTCCD (1,0,XC(1),YC(1),2)                                    PLTVG20
      XC(1)=XP-.25                                                       PLTVG21
      XC(2)=XP+.25                                                       PLTVG22
      YC(1)=YP                                                           PLTVG23
      YC(2)=YP                                                           PLTVG24
    2 CALL PLTCCD (1,0,XC(1),YC(1),2)                                    PLTVG25
    3 CONTINUE                                                           PLTVG26
      CALL PLTCCT (.15,TX(1),0.,1.,4.,.75)                               PLTVG27
      CALL PLTCCT (.15,TY(1),1.,0.,.9,3.)                                PLTVG28
      CALL PLTCCT (.10,ITL(1),0.,1.,1.5,.5)                              PLTVG29
      XB=XB+1.5                                                          PLTVG30
```

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      YB=YB+1.5                                              PLTVG31
      CALL PLTCCS (XB,YB,XOR,YOR,XS,YS)                      PLTVG32
      XMAX=XOR+8.*XS                                         PLTVG33
      YMAX=YOR+6.*YS                                         PLTVG34
      CALL PLTCCA (XS,YS,XOR,XMAX,YOR,YMAX,0)                PLTVG35
      CALL PLTCCD (M,NS,X(1),Y(1),N)                         PLTVG36
      CALL LABELA (XS,YS,XOR,XMAX,YOR,YMAX,1.,1.)            PLTVG37
      RETURN                                                 PLTVG38
      END                                                    PLTVG39-
*     COMPILE DISC,GENLSQ,ALL
*     COMPILE DISC,SORTXY,ALL
*     COMPILE DISC,LABELA,ALL
C                                                                      END
```

TABLE A-1

.45 GM TUNGSTEN SPHEROID INTO .15 CM ALUMINUM AT 0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 243.8 TO 310.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	243.8	.246400	.328262	
FINAL ESTIMATES	298.0	.504504	.222161	13.0

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
243.8	.0		
302.4	153.0	152.1	1.0
355.7	246.3	253.0	-6.7
363.3	271.0	262.0	9.0
373.4	253.3	273.4	-20.1
374.9	266.4	275.1	-8.7
399.9	321.3	302.2	19.1
422.8	318.2	325.9	-7.7
976.9	880.9	862.8	18.0
1026.3	905.9	910.4	-4.6
1033.3	921.7	917.2	4.5
1036.6	940.3	920.4	19.9
1494.1	1371.6	1363.3	8.3
1501.4	1353.0	1370.4	-17.4
1547.2	1400.6	1414.9	-14.3

STRIKING VELOCITY M/S

.45 GM TUNGSTEN SPHEROID INTO .15 CM ALUMINUM AT 0 DEGREES OBLIQUITY

FIGURE A-1

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.45 GM TUNGSTEN SPHEROID INTO .32 CM
ALUMINUM AT 0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 343.5
TO   460.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	343.5	.426603	.214394	
FINAL ESTIMATES	343.5	.333039	.253525	40.7

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
343.5	.0		
456.9	379.5	305.6	73.9
526.4	307.8	375.2	-67.3
623.3	423.1	468.0	-44.9
899.2	718.7	727.1	-8.3
915.3	766.0	742.2	23.7
920.8	770.5	747.4	23.2
1524.0	1326.2	1318.0	8.1
1531.6	1322.2	1325.3	-3.1

STRIKING VELOCITY M/S

.45 GM TUNGSTEN SPHEROID INTO .32 CM ALUMINUM AT 0 DEGREES OBLIQUITY

FIGURE A-2

RESIDUAL VELOCITY M/S

TABLE A-2

.45 GM TUNGSTEN SPHEROID INTO   .32 CM
ALUMINUM AT   0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM   343.5
TO   460.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	343.5	.426603	.214394	
FINAL ESTIMATES	343.5	.333039	.253525	40.7

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
343.5	.0		
456.9	379.5	305.6	73.9
526.4	307.8	375.2	-67.3
623.3	423.1	468.0	-44.9
899.2	718.7	727.1	-8.3
915.3	766.0	742.2	23.7
920.8	770.5	747.4	23.2
1524.0	1326.2	1318.0	8.1
1531.6	1322.2	1325.3	-3.1

TABLE A-3

.45 GM TUNGSTEN SPHEROID INTO .15 CM
MILD STEEL AT 0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 343.5 TO 380.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	343.5	.213101	.292740	
FINAL ESTIMATES	358.1	.291155	.244159	56.8

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
281.6	.0		
305.4	.0		
318.5	.0		
334.1	.0		
343.5	.0		
365.5	138.4	138.0	.3
388.3	183.8	189.5	-5.7
391.4	204.2	194.1	10.1
410.3	208.2	219.3	-11.1
559.3	370.9	366.1	4.8
883.6	711.4	653.2	58.2
920.8	710.2	686.0	24.2
920.8	696.8	686.0	10.8
961.6	780.0	722.2	57.8
989.7	755.3	747.0	8.3
1026.3	596.5	779.5	-183.0
1524.0	1228.0	1226.2	1.9
1531.6	1257.9	1233.1	24.8

STRIKING VELOCITY M/S

.45 GM TUNGSTEN SPHEROID INTO .15 CM MILD STEEL AT 0 DEGREES OBLIQUITY

FIGURE A-3
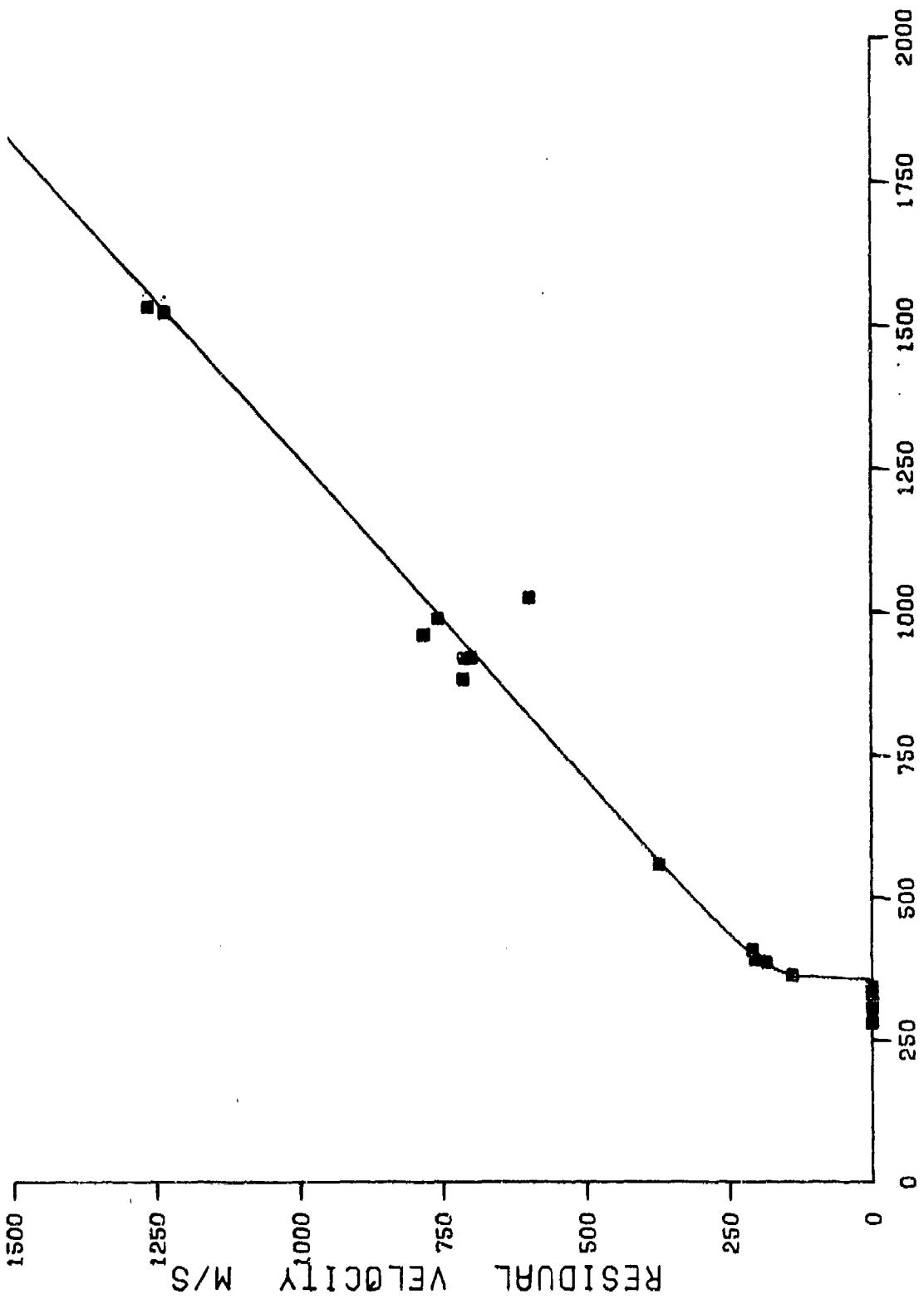
TABLE A-4

.45 GM TUNGSTEN SPHEROID INTO .32 CM
MILD STEEL AT 0 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 515.7
TO 530.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	515.7	.108057	.324210	
FINAL ESTIMATES	515.7	.089492	.356297	29.0

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
515.7	.0		
520.9	92.4	77.5	14.9
598.9	229.2	210.3	18.9
603.5	189.0	215.1	-26.2
607.2	239.0	219.0	20.0
873.3	389.5	451.2	-61.6
909.8	511.1	481.2	29.9
934.8	502.9	501.7	1.2
1479.5	942.1	954.0	-11.8
1516.4	1025.7	985.2	40.4
1531.6	982.1	998.2	-16.1

STRIKING VELOCITY M/S

.45 GM TUNGSTEN SPHEROID INTO .32 CM MILD STEEL AT 0 DEGREES OBLIQUITY

FIGURE A-4
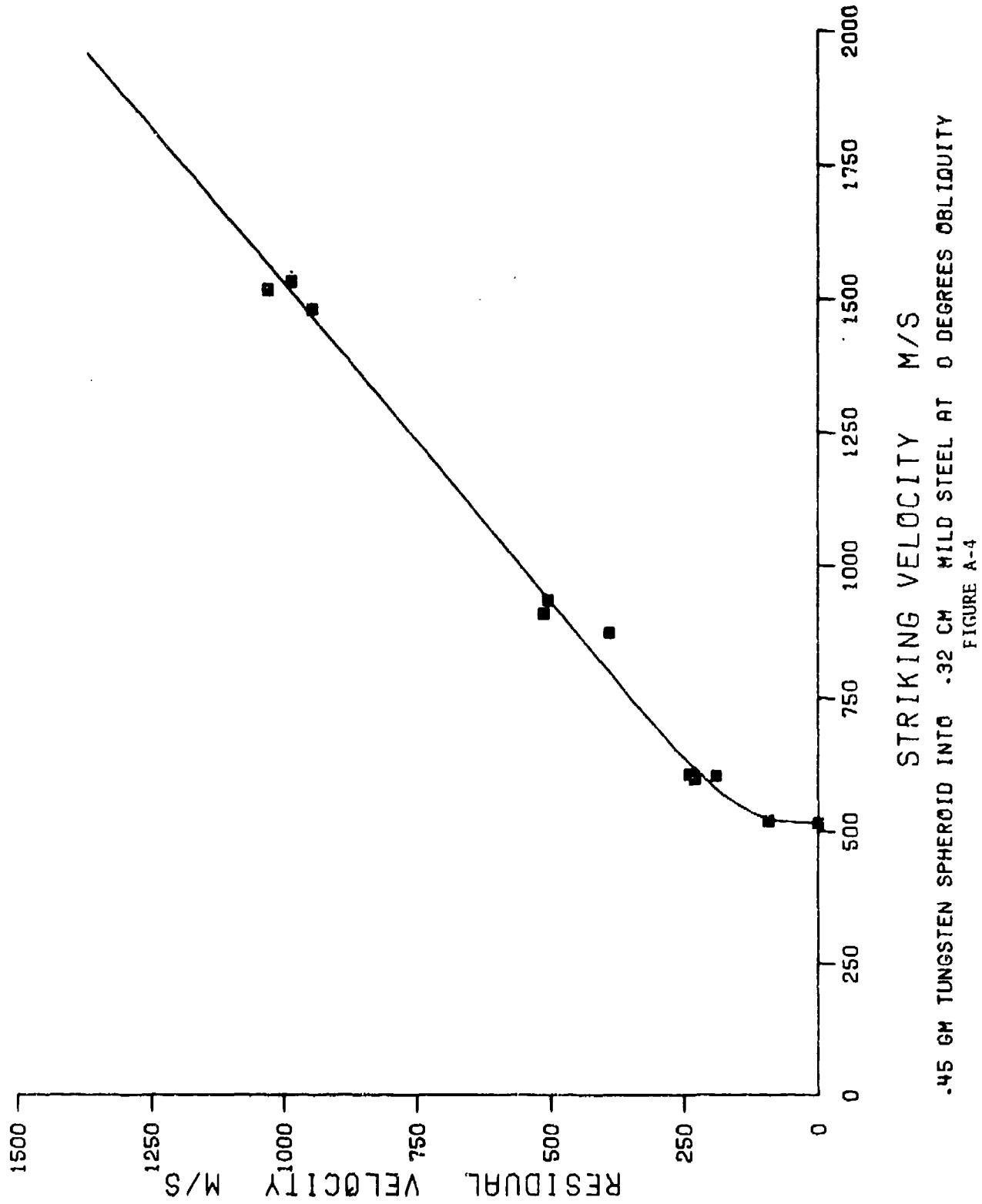
TABLE A-5

.45 GM TUNGSTEN SPHEROID INTO .15 CM
ALUMINUM AT 45 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 266.4
TO 410.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	266.4	.034665	.593401	
FINAL ESTIMATES	266.4	.028704	.623741	62.9

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
266.4	.0		
404.8	226.5	187.3	39.1
468.8	297.2	255.6	41.6
480.7	289.0	268.2	20.8
507.2	254.2	296.4	-42.2
584.9	238.4	379.3	-140.9
907.1	804.7	726.9	77.7
996.1	868.4	823.5	44.9
1417.6	1285.0	1279.7	5.4
1571.2	1397.8	1444.7	-46.9

STRIKING VELOCITY M/S

.45 GM TUNGSTEN SPHEROID INTO .15 CM ALUMINUM AT 45 DEGREES OBLIQUITY
FIGURE A-5

RESIDUAL VELOCITY M/S

48
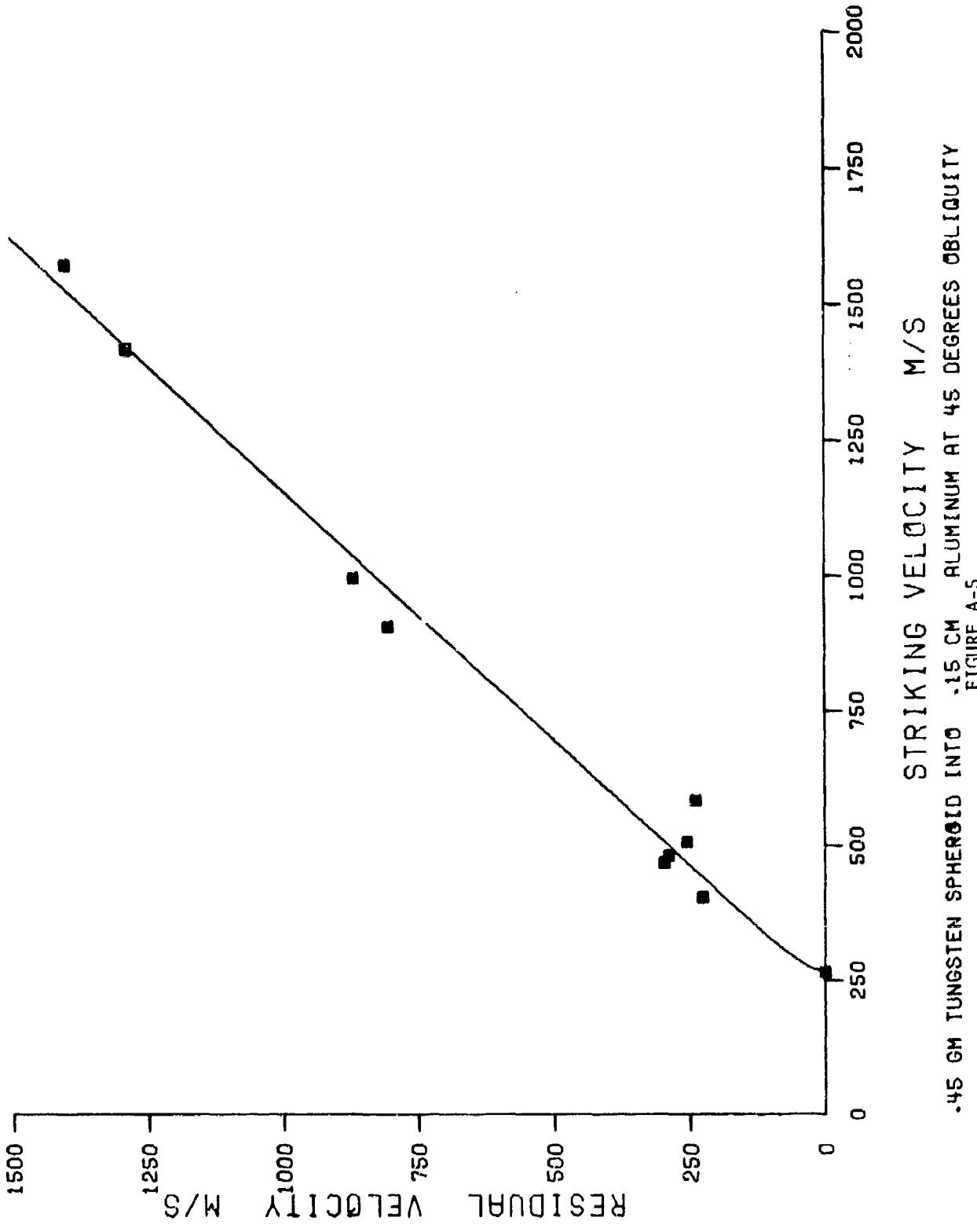
TABLE A-6

.45 GM TUNGSTEN SPHEROID INTO .32 CM
ALUMINUM AT 45 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 482.2
TO 600.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	482.2	.114710	.390246	
FINAL ESTIMATES	581.7	.344483	.226746	43.2

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
482.2	.0		
597.7	291.7	284.4	7.3
666.9	383.4	407.4	-23.9
695.9	395.6	442.0	-46.4
880.9	663.9	629.7	34.2
912.6	695.9	659.8	36.1
952.5	727.9	697.4	30.5
1516.4	1179.0	1217.7	-38.7
1531.6	1166.2	1231.7	-65.5
1555.1	1322.8	1253.3	69.5

STRIKING VELOCITY M/S
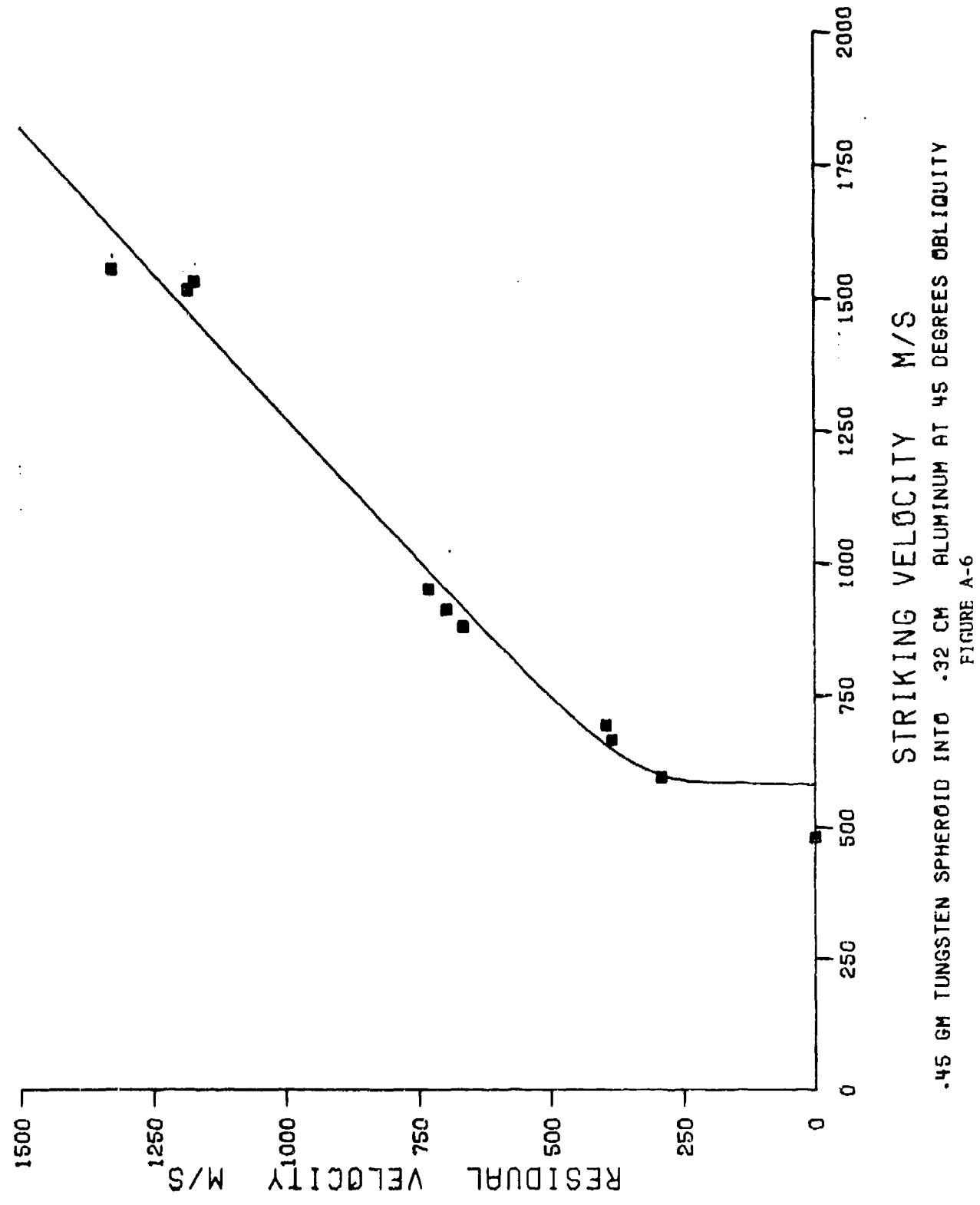
.45 GM TUNGSTEN SPHEROID INTO .32 CM ALUMINUM AT 45 DEGREES OBLIQUITY

FIGURE A-6

TABLE A-7

### .45 GM TUNGSTEN SPHEROID INTO .15 CM
MILD STEEL AT 45 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM 352.3 TO 550.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	352.3	.273901	.247218	
FINAL ESTIMATES	526.7	.604598	.132986	30.5

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
352.3	.0		
541.3	316.7	313.1	3.6
594.1	408.1	387.9	20.2
613.3	379.8	407.9	-28.2
623.3	356.6	418.0	-61.3
658.4	430.4	451.4	-21.0
662.6	498.0	455.3	42.7
686.4	477.9	477.0	1.0
705.6	501.4	494.2	7.2
718.7	518.5	505.8	12.7
734.6	542.2	519.7	22.5
912.6	685.8	672.4	13.4
923.5	716.6	681.7	34.9
923.5	681.8	681.7	.2
949.5	723.3	703.6	19.7
1015.9	697.1	759.7	-62.6
1508.8	1178.1	1175.9	2.1
1516.4	1153.1	1182.4	-29.3
1524.0	1239.0	1188.8	50.2
1539.5	1176.2	1202.0	-25.8

STRIKING VELOCITY M/S
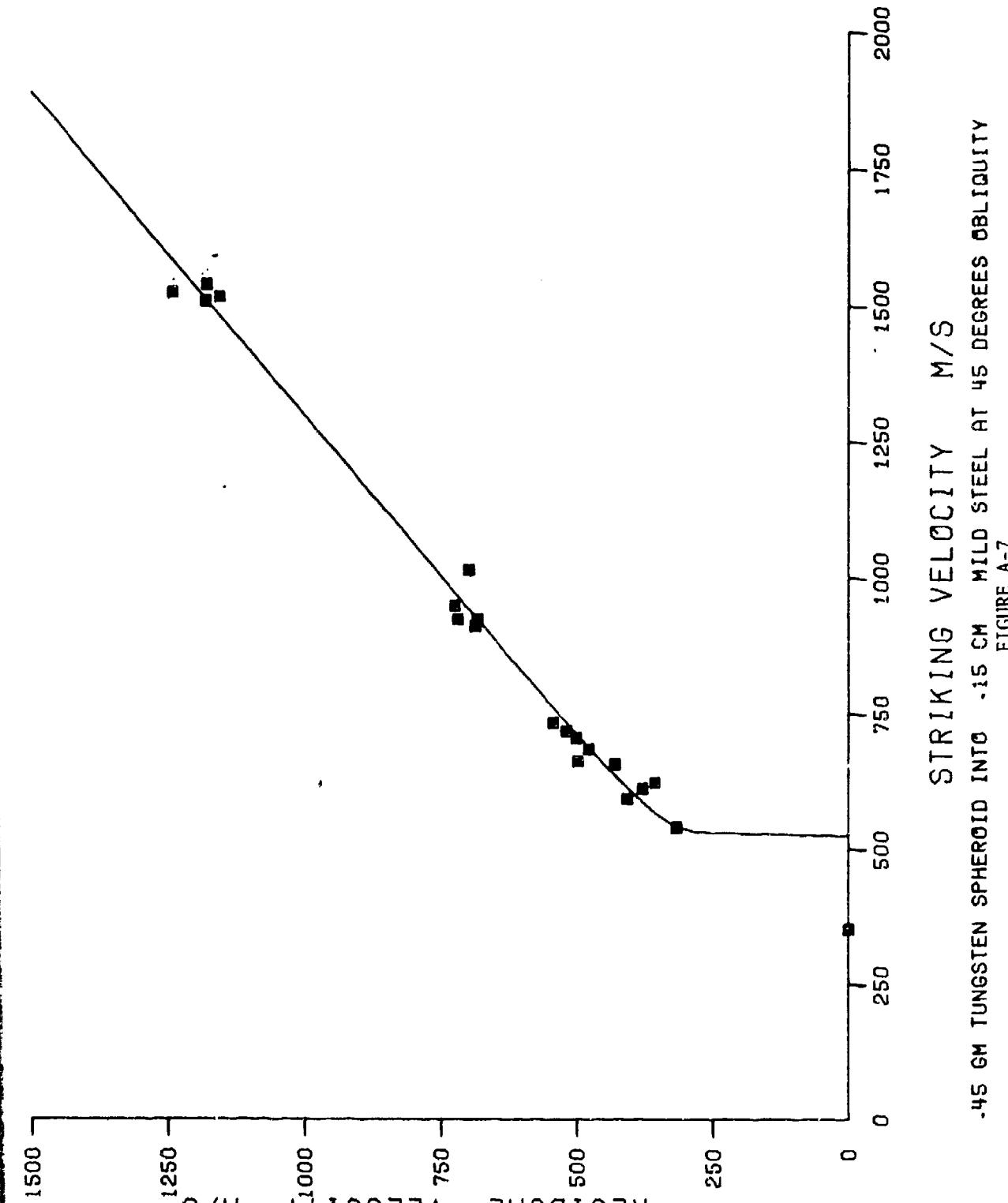
.45 GM TUNGSTEN SPHEROID INTO .15 CM MILD STEEL AT 45 DEGREES OBLIQUITY

FIGURE A-7

RESIDUAL VELOCITY M/S

S2

.45 GM TUNGSTEN SPHEROID INTO   .32 CM
MILD STEEL AT 45 DEGREES OBLIQUITY

CRITICAL VELOCITY IS CONSTRAINED TO THE INTERVAL FROM   800.1
TO   890.0 M/S.

WEIBULL MODEL	VC M/S	LAMBDA	BETA	ERMS
INITIAL ESTIMATES	800.1	.207160	.200348	
FINAL ESTIMATES	877.6	.396242	.101630	58.9

STRIKING VELOCITY M/S	RESIDUAL VELOCITY M/S	APPROXIMATION M/S	ERROR M/S
800.1	.0		
880.9	317.6	317.8	-.2
955.5	454.2	439.9	14.2
979.9	447.4	460.2	-12.8
1516.4	843.7	810.0	33.7
1516.4	695.6	810.0	-114.5
1539.5	903.4	824.4	79.0

STRIKING VELOCITY M/S

.45 GM TUNGSTEN SPHEROID INTO .32 CM MILD STEEL AT 45 DEGREES OBLIQUITY
FIGURE A-8

54
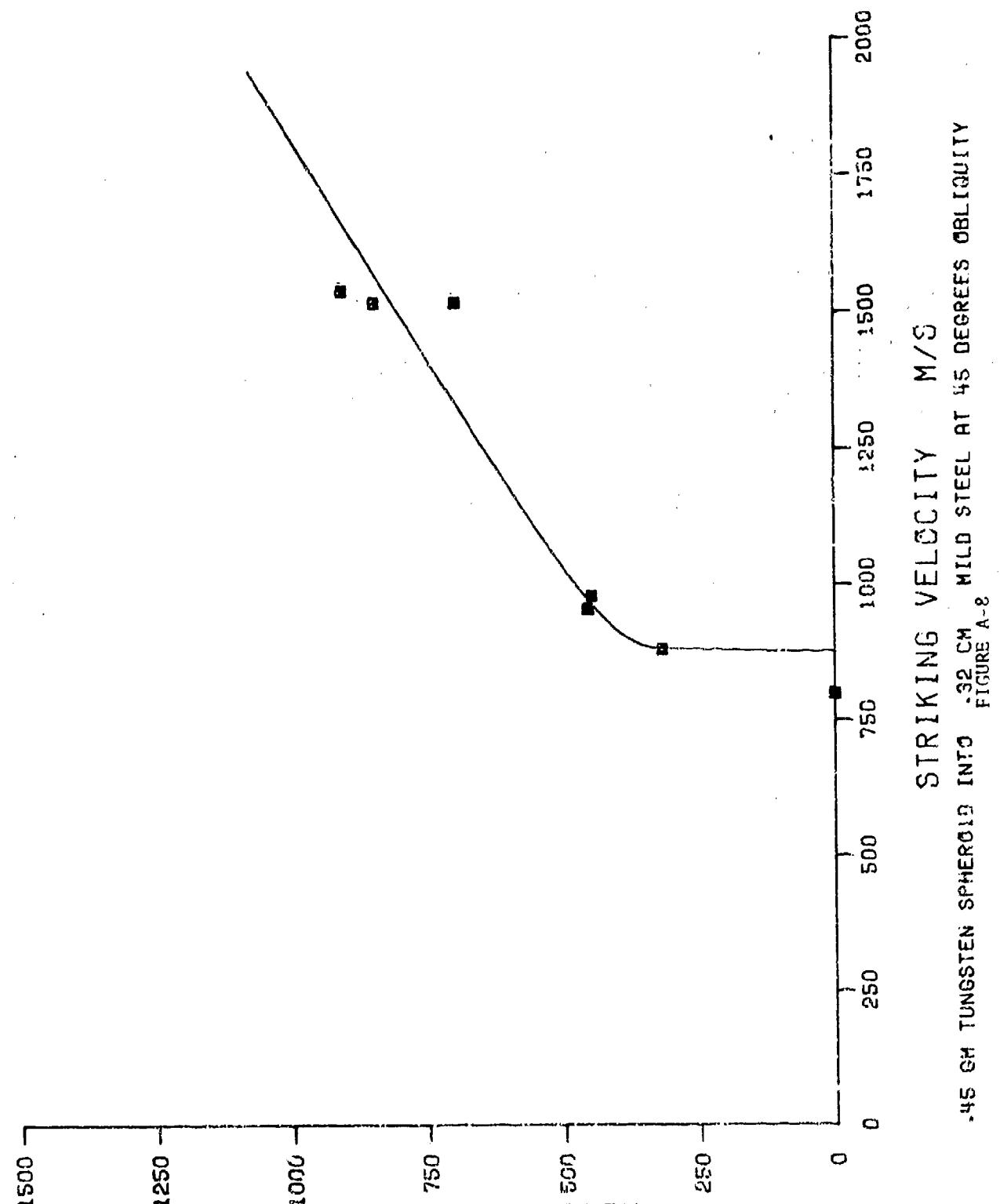
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